

THIS REPORT HAS BEEN DELIMITED
AND CLEARED FOR PUBLIC RELEASE
UNDER DOD DIRECTIVE 5200.20 AND
NO RESTRICTIONS ARE IMPOSED UPON
ITS USE AND DISCLOSURE.

DISTRIBUTION STATEMENT A

APPROVED FOR PUBLIC RELEASE,
DISTRIBUTION UNLIMITED.

Armed Services Technical Information Agency

Because of our limited supply, you are requested to return this copy WHEN IT HAS SERVED YOUR PURPOSE so that it may be made available to other requesters. Your cooperation will be appreciated.

AD

45690

NOTICE: WHEN GOVERNMENT OR OTHER DRAWINGS, SPECIFICATIONS OR OTHER DATA ARE USED FOR ANY PURPOSE OTHER THAN IN CONNECTION WITH A DEFINITELY RELATED GOVERNMENT PROCUREMENT OPERATION, THE U. S. GOVERNMENT THEREBY INCURS NO RESPONSIBILITY, NOR ANY OBLIGATION WHATSOEVER; AND THE FACT THAT THE GOVERNMENT MAY HAVE FORMULATED, FURNISHED, OR IN ANY WAY SUPPLIED THE SAID DRAWINGS, SPECIFICATIONS, OR OTHER DATA IS NOT TO BE REGARDED BY IMPLICATION OR OTHERWISE AS IN ANY MANNER LICENSING THE HOLDER OR ANY OTHER PERSON OR CORPORATION, OR CONVEYING ANY RIGHTS OR PERMISSION TO MANUFACTURE, USE OR SELL ANY PATENTED INVENTION THAT MAY IN ANY WAY BE RELATED THERETO.

Reproduced by
DOCUMENT SERVICE CENTER
KNOTT BUILDING, DAYTON, 2, OHIO

UNCLASSIFIED

AD No. 45690

ASTIA FILE COPY

TECHNICAL REPORT NO. 6418-8

AN INEXPENSIVE SUPERSONIC
WIND TUNNEL FOR HEAT-TRANSFER
MEASUREMENTS

PART II—RESULTS FOR A LAMINAR BOUNDARY LAYER BASED ON A TWO-DIMENSIONAL
FLOW MODEL FOR THE ENTRANCE REGION OF A TUBE

BY

JOSEPH KAYE AND GEORGE A. BROWN

FOR

OFFICE OF NAVAL RESEARCH

CONTRACT N5ori-07805

NR-061-028

D. I. C. PROJECT NUMBER 6418

OCTOBER 1, 1954

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

DEPARTMENT OF MECHANICAL ENGINEERING

AND

DIVISION OF INDUSTRIAL COOPERATION

CAMBRIDGE 39, MASSACHUSETTS

AN INEXPENSIVE SUPERSONIC WIND TUNNEL
FOR HEAT-TRANSFER MEASUREMENTS

Part II - Results for a Laminar Boundary Layer Based on a Two-
Dimensional Flow Model for the Entrance Region of a Tube.

By

Joseph Kaye¹ and George A. Brown²

SUMMARY

Reliable experimental data on local heat-transfer coefficients for supersonic flow of air in a round tube are reanalyzed in detail with the aid of an approximate two-dimensional flow model. The results are compared with similar results based on a one-dimensional flow model and with the theoretical predictions for supersonic flow over a flat plate and for flow in the entrance region of a tube when a laminar boundary layer is present.

The two-dimensional flow model yields a better understanding of the phenomena which occur for diabatic supersonic flow of air in a round tube than that obtained with the aid of the one-dimensional flow model. The two-dimensional flow model shows that the core Mach number is nearly constant along the length of test section for a range of values of the inlet diameter Reynolds number. For a laminar boundary layer, the values of the local Stanton number agree within a few per cent with the theoretical values for plate flow at the largest values of the inlet diameter Reynolds number.

¹ Associate Professor of Mechanical Engineering, Massachusetts Institute of Technology.

² Shell Fellow in Mechanical Engineering, Massachusetts Institute of Technology.

NOMENCLATURE

a	- inside radius of pipe
A	- cross-sectional area, $\pi D^2/4$
A'	- heat-transfer area, πDAL
c_p	- specific heat at constant pressure
c_w	- discharge coefficient of nozzle
D	- inside diameter of pipe
g	- acceleration given to unit mass by unit force
G	- flow per unit area, w/A
h	- coefficient of heat transfer, $q/A'(t_w - t_{aw})$
k	- ratio of specific heats
L	- distance from end of curved contour of nozzle
M	- Mach number, V/\sqrt{gkRT}
n	- summation index, Eq. (15)
Nu_L	- length Nusselt number, hL/λ
p	- static pressure
q	- net rate of heat transfer
R	- perfect gas constant
Re_D	- diameter Reynolds number, $DG/\mu g$
Re_L	- length Reynolds number, $LG/\mu g$
St	- Stanton number, $h/c_p G$
t	- temperature, deg F
T	- temperature, deg F abs
V	- velocity
w	- mass rate of flow
ρ	- density
μ	- viscosity
λ	- thermal conductivity
δ	- thickness of boundary layer

Superscript * refers to throat of supersonic nozzle where $M = 1$.

Subscripts:

aw	- adiabatic wall conditions
b	- boundary layer
c	- isentropic core
j	- station numbers
o	- hypothetical entrance plane of the tube, where the boundary layer is of zero thickness
ob	- stagnation conditions in boundary layer
oc	- stagnation conditions in core
oi	- upstream stagnation conditions
r	- atmospheric conditions
s	- isentropic conditions
w	- wall conditions
∞	- free stream conditions for flat-plate flow

INTRODUCTION

An inexpensive steady-state supersonic wind tunnel has been designed for accurate measurements of local heat-transfer coefficients. It consists of a convergent-divergent nozzle which delivers dry air at supersonic speeds to a test section, one-half inch in diameter and 30 to 50 diameters in length. The various test apparatus and the measurements obtained therefrom have been described previously in detail (1).^{*} Sufficient measurements have been made to demonstrate their reliability when a laminar boundary layer exists in the test section for adiabatic and diabatic supersonic flow. The heat-transfer data were first analyzed (1) with the aid of a simple one-dimensional flow model, abbreviated hereafter as 1-DFM, but it was shown that this model is not fully adequate to explain, interpret, and correlate these measurements.

The value of these measurements of local heat-transfer coefficients would be considerably enhanced if they could be compared with similar measurements for other types of supersonic flow with a laminar boundary layer, such as plate flow, cone flow, etc. One reason for subjecting the data given in (1) to further analysis and computation is to lay the foundation for such comparisons. A second reason is that accurately measured values of local heat-transfer coefficients for supersonic flow with a laminar boundary layer are practically nonexistent so that further analysis of tube-flow data might produce information useful to the designer of devices moving at supersonic speeds. A third reason for using a more complicated flow model to reanalyze the same data is to provide the experimenter with a better phenomenological explanation of the processes which occur in supersonic flow in the entrance region of a tube. The two-dimensional flow model, abbreviated hereafter as 2-DFM, will be used in this paper to analyze and interpret some selected values of the data given in (1).

The analysis of supersonic flow in the inlet region of a tube can be accomplished, on the one hand, in an extremely simple fashion, such as with the aid of the 1-DFM, and on the other hand, in an extremely difficult fashion, such as by investigation of the partial differential equations of energy, momentum, and continuity. Although both of these methods of analysis have been used in the present research program, an intermediate type of analysis is used here to obtain results close to those based on the most exact analysis. This 2-DFM should be considered to represent closely the actual supersonic flow phenomena in the tube.

The 2-DFM is used here only for supersonic flow in the entrance region of a tube with a laminar boundary layer originating at tube inlet. Experimentally it has been found possible to maintain a laminar boundary layer over the entire length of test section by careful control of the inlet conditions. Independent measurements of the velocity and temperature profiles of the tube flow for these inlet conditions have confirmed the existence of this laminar boundary layer. In the previous paper (1) certain values of the tube inlet conditions were selected to present the data for a laminar boundary layer. In

^{*}Numbers in parentheses refer to the Bibliography at the end of the paper.

the present paper, the same values of these inlet conditions are selected to present the results of computation based on the 2-DFM. Comparisons of the results based on the 1-DFM and on the 2-DFM, for the same original data, will be given here. The results based on the 2-DFM will also be compared with theoretical solutions for plate flow and for tube flow.

ASSUMPTIONS FOR TWO-DIMENSIONAL FLOW MODEL WITH HEAT TRANSFER

The 2-DFM arbitrarily divides the supersonic flow in the entrance region of a tube into two parts, a laminar boundary layer and a central core of fluid, as shown in Fig. 1. The laminar boundary layer originates near the tube inlet and grows in thickness by transference of mass from the central core. This 2-DFM is consistent with Prandtl's concept of the boundary layer since it is assumed that both the viscous and thermal effects are concentrated in the boundary layer, and that both are negligible in the core region, i.e., the fluid remaining in the central core undergoes an isentropic process. The basic soundness of this 2-DFM has been conclusively demonstrated by means of accurately measured velocity profiles at various positions along the test section of Fig. 1, for both adiabatic and diabatic flow (2, 3), and also by means of unpublished recently measured temperature profiles.

The present paper deals with a simplified version of the actual 2-DFM in that the actual velocity and temperature profiles are represented by simple approximations for both the laminar boundary layer and the core. The boundary-layer thickness is assumed to be the same for both the velocity and thermal boundary layers. This thickness is arbitrarily taken to correspond to that thickness at which the local velocity in the boundary layer equals 99 per cent of the mean velocity in the core. These simplifications are shown in Fig. 1. They were introduced mainly to reduce the computational time since it was found, after considerable study, that the use of these simplified velocity and temperature profiles yielded results nearly identical to those obtained using a non-linear velocity profile but requiring considerably greater computational time.

The following assumptions are made in the analysis of the 2-DFM:

1. A laminar boundary layer, in which both the viscous and thermal effects are predominant, exists for supersonic flow in the entrance region.
2. The air which remains in the central core undergoes an isentropic change of state from an upstream stagnation state to the static pressure measured at a given section.
3. The properties in the central core of fluid are assumed to be uniform at a given section.

4. The static pressure is assumed to be uniform across each section of the tube.

5. Air is a perfect gas with a constant value of the ratio of specific heats ($k = 1.40$) over the range of temperature under consideration.

6. The laminar boundary layer has a linear distribution of velocity which is further approximated by an average velocity in the boundary layer equal to one-half the core velocity, as shown in Fig. 1.

7. The stagnation temperature of the central core is assumed to be constant at all sections, equal to T_{oc} , whereas the stagnation temperature of the boundary layer is assumed to be uniform at a given section but variable from section to section.

8. The flow is adiabatic up to the tube inlet where the heat transfer originates.

ANALYSIS OF SIMPLIFIED TWO-DIMENSIONAL FLOW MODEL: WITH HEAT TRANSFER

The following relations hold at each section of the flow in the tube shown in Fig. 1.

$$\text{Continuity:} \quad w = w_b + w_c \quad (1)$$

$$\text{Continuity:} \quad w_c = \rho_c V_c A_c \quad (2)$$

$$\text{Continuity:} \quad w_b = \rho_b V_b A_b = \rho_b V_b (A - A_c) \quad (3)$$

$$\text{Geometry:} \quad A_c = \pi \left[(D/2) - \delta \right]^2 \quad (4)$$

$$\text{Geometry:} \quad A = A_b + A_c \quad (5)$$

$$\text{Equation of state:} \quad p = \rho_c R T_c \quad (6)$$

$$\text{Equation of state:} \quad p = \rho_b R T_b \quad (7)$$

$$\text{Definition:} \quad M_c^2 = V_c^2 / g k R T_c \quad (8)$$

$$\text{Assumption:} \quad V_b = V_c / 2 \quad (9)$$

The following relations hold between the upstream stagnation state and a state in the core at any tube section:

$$\text{Isentropic:} \quad p_{oi}/p = (\rho_{oi}/\rho_c)^k \quad (10)$$

$$\text{Isentropic:} \quad T_{oi}/T_c = (p_{oi}/p)^{(k-1)/k} \quad (11)$$

$$\text{Isentropic:} \quad T_{oi} = T_{oc} \quad (12)$$

$$\text{Isentropic:} \quad c_p T_{oc} = c_p T_c + V_c^2 / 2g \quad (13)$$

Definition: $c_p T_{ob} = c_p T_b + V_b^2/2g$ (14)

Application of the energy equation for steady flow to a control volume which encloses the fluid between the upstream stagnation region and any tube section j yields

$$(1/2)q_j + \sum_{n=1}^{n=j-1} q_n = w_{cj} (c_p T_{cj} + V_c^2/2g) + w_{bj} (c_p T_{bj} + V_b^2/2g) - w c_p T_{oi}. \quad (15)$$

The summation in Equation (15) starts at the first station at which heat transfer data were measured. This first station for test combinations C and D is described in reference (i).

The additional relations used in this analysis are as follows: The condition for choked flow of air through the supersonic nozzle yields

$$G_s^* = (w/A^*)_s = 0.5318 p_{oi} / \sqrt{T_{oi}} \quad (16)$$

Definition: $c_w = (w/A^*)/(w/A^*)_s$ (17)

Assumption: $T_{aw}/T_{oi} = 0.940$ (18)

Definition: $h = q/A' (t_w - t_{aw})$ (19)

Definition: $Nu_{Dc} = hD/\lambda_c$ (20)

Definition: $Nu_{Lc} = hL/\lambda_c$ (21)

Definition: $St_c = h/c_p \rho_c V_c$ (22)

Definition: $Re_{Dc} = \rho_c V_c D_c / \mu_c g$ (23)

Definition: $Re_{Lc} = \rho_c V_c L / \mu_c g$ (24)

Method of Computation

A sample calculation is given in the Appendix.

RESULTS

General

The original measurements of heat-transfer coefficients for supersonic flow obtained with test combinations C and D, comprising four runs with the former and thirteen runs with the latter, are given in detail in (1). These data have been selected for those stagnation conditions where a laminar boundary layer is present over most of the test section. Hence, the method of computation outlined above for the simplified 2-DFM should be applicable for these seventeen runs. In view of the large amount of calculation involved, however, only seven runs have been studied in detail by means of the simplified 2-DFM, namely six and one from test combinations C and D, respectively.

The results for six runs made with test combination D are given in Figs. 2 to 7, inclusive. Six charts were found necessary to present these results in sufficient detail because they are sensitive to small changes in the value of the inlet diameter Reynolds number. The results for one run made with test combination C are given in Fig. 8. One run was selected here mainly to reduce the total computational time and to show the effect of a shorter test section on the data. On each of these seven charts the calculated quantities based on one flow model are plotted without reference to the quantities calculated from the other flow model. Thus, in the curve of Mach number versus length Reynolds number, the values of Mach number and Reynolds number are taken from either the 1-DFM or from the 2-DFM.

Each of the seven charts presents the Mach number, boundary-layer thickness ratio, local length Nusselt number, and local Stanton number. For the 2-DFM, the Mach number, Nusselt number, and Stanton number are based on properties of the central core of fluid. The local Nusselt and Stanton numbers are, in reality, average values over the short length of flow of two or four pipe diameters. In the discussion of these seven charts, the value of the inlet diameter Reynolds number chosen for identification corresponds to that computed by means of the 2-DFM.

Each of the seven charts contains a comparison of the results based on the 2-DFM with the theoretical predictions of supersonic flow with a laminar boundary layer on a flat plate and with the theoretical predictions for tube flow for constant values of the viscosity and the thermal conductivity. The former predictions are taken from van Driest (4) and the latter from Toong (5). This type of comparison of results based on the 2-DFM with predictions for plate flow and for tube flow should be regarded as a temporary expedient until exact solutions for tube flow and accurate experimental data for plate flow with adverse pressure gradients are available.

Results for Test Combination D-Mach Number

Fig. 2 presents the results based on the simplified 2-DFM for the lowest value of the inlet diameter Reynolds number, namely, 33,000. The Mach number

based on the 2-DFM decreased only from 2.7 to 2.1 along the length of the test section whereas that for the 1-DFM decreases from 2.1 to about 1.2. This same type of behaviour of the Mach numbers for the two flow models is found also in Figs. 3, 4, 5, 6, and 7. In Fig. 7, the Mach number based on the 2-DFM decreased only from 2.65 to 2.50 over 50 diameters of length of flow. This constancy of the Mach number based on the 2-DFM supports the contention that this type of test section is, in reality, a useful and inexpensive type of steady-state supersonic wind tunnel. The 2-DFM shows that the core Mach number does not vary as much as that indicated by the 1-DFM and that the value of the core Mach number is significantly greater than the corresponding value based on the 1-DFM. Independent measurements of velocity and temperature profiles at several stations near the tube exit lead to calculated values of the core Mach number which agree, within one per cent, with the values of the core Mach number based on the 2-DFM.

Boundary-Layer Thickness

The boundary-layer thickness at any section in the tube is defined by Equation (4) for the 2-DFM. It should be noted that this thickness is taken arbitrarily as zero in the 1-DFM. For the smaller values of the inlet diameter Reynolds number, namely from 33,000 to 72,000 in Figs. 2, 3, and 4, this ratio increased from about 20 per cent at tube inlet to about 40 to 50 per cent at tube exit. For the larger values of Reynolds number, namely from 88,000 to 115,000 in Figs. 5, 6, and 7, this ratio increases from about 10 per cent at tube inlet to about 25 per cent at tube exit. Such behaviour of the thickness ratio could be interpreted in two ways. Either the laminar boundary layer does not fill the tube at exit and a fully developed flow pattern is not attained for this entrance type of flow, or for the flow length of 50 diameters the supersonic flow is, in reality, one corresponding to a fully developed flow but with a velocity profile which is quite different than that for fully developed incompressible flow in a round tube. Additional experimental data on velocity profiles for several stations near the tube exit are needed before these views can be resolved.

The thickness ratio for tube flow based on the 2-DFM can be compared with a similar ratio derived for the thickness of a laminar boundary layer of a compressible fluid flowing on a flat plate for zero pressure gradient. The theoretical results of Howarth (6) lead to the following equation for the thickness ratio,

$$(\delta/a) = 10.4(L/D)(1 + 0.0795 M_c^2)/(Re_{Lc})^{1/2}. \quad (25)$$

Here the thickness δ corresponds to that value at which the local velocity in the boundary layer on the flat plate is 99 per cent of the free stream value. The values of the thickness ratio based on the 2-DFM are slightly larger than those based on Equation (25) for values of the diameter Reynolds number from

33,000 to 72,000, but both sets of values are in excellent agreement over the entire length of the test section for Reynolds numbers from 88,000 to 115,000. This excellent agreement should be considered as somewhat coincidental for the following reasons:

1. The zero of the length measurement for the tube flow should be the origin of the boundary layer where its thickness is zero, but the actual zero has been arbitrarily taken to be at the end of the curved contour of the nozzle. The effects introduced by this error should disappear by the middle of the test section.
2. The Howarth formula is restricted to zero pressure gradient whereas a finite adverse pressure gradient exists in the tube flow. The smallest pressure gradient is found, however, at the largest value of the inlet diameter Reynolds number, and for this case, the agreement between thickness ratio based on the 2-DFM and on the Howarth formula is the best.
3. At the lowest values of the diameter Reynolds number, namely at 33,000 to 72,000, some evidence exists that separation of the laminar boundary layer probably occurs in the presence of the adverse pressure gradient. Such separation would probably result in a thickening of the laminar boundary layer and cause its calculated thickness based on the 2-DFM to be greater than that indicated by the Howarth formula. Such a thickening calculated on the basis of the 2-DFM, can be seen in Figs. 2, 3, and 4.

Stanton Number

The local heat-transfer coefficients based on the 2-DFM are shown in Figs. 2 to 7 for test combination D in the form of the local Stanton and local length Nusselt numbers. Comparison of the Stanton numbers shows that the major effect of shifting from the 1-DFM to the 2-DFM is to decrease the calculated values of the Stanton numbers and simultaneously to increase the corresponding values of the length Reynolds number. In Fig. 2, for a diameter Reynolds number of 33,000, the first six values of the Stanton number lie within an average deviation of less than 10 per cent from the theoretical solution for tube flow, the remaining eight values fluctuate about a mean value, and the fluctuations begin at a length Reynolds number based on the 2-DFM of 600,000. Similar fluctuations are evident in the values of the Stanton number based on the 1-DFM and are discussed in (1). A similar process of levelling-off of the values of the Stanton number with fluctuations about a mean value is evident in Figs. 3 and 4 at larger values of the diameter Reynolds number.

Figs. 2 to 7 indicate that as the value of the diameter Reynolds number increases from 33,000 to 115,000 the values of the Stanton number based on the 2-DFM move from below the lower line for the tube solution to almost perfect agreement with the theoretical plate-flow solution, shown in Fig. 7. The discrepancies between the values of the Stanton number and the tube solution should not be considered significant at present, since this tube solution

is based on the assumption of constant viscosity and thermal conductivity; work is in progress to obtain tube solutions for variable values of these properties. The discrepancies between the values of the Stanton number and the flat-plate solution, especially at the smaller values of the diameter Reynolds number should not be considered significant for two reasons. First, the plate-solution is based on zero pressure gradient but the data were taken with an adverse pressure gradient in the tube; note that the best agreement between the tube-flow data and the flat-plate solution occurs for the smallest value of the adverse pressure gradient in the tube. Second, at the smallest values of the diameter Reynolds number, there is some evidence of separation of the laminar boundary layer so that one would expect some deviations from the theoretical solution for plate flow in which separation effects were neglected.

Nusselt Number

The values of the local length Nusselt number based on the 2-DFM and on the 1-DFM are also shown in Figs. 2 to 7. The main reason for including the Nusselt number is to compare the value of plotting the calculations in different ways. All the conclusions reached so far with the aid of the curves of Stanton number are also evident from examination of curves for the Nusselt number, except that the levelling-off of the Stanton numbers in Figs. 2, 3, and 4 is almost completely masked in the sharply rising values of the Nusselt number at the same length Reynolds number. This masking is a result of using the tube length as a multiplicative factor in the Nusselt number.

Transition

The transition from a laminar to a turbulent boundary layer is clearly evident in Figs. 5, 6, and 7, occurring as a very sharp rise in value of the Stanton or Nusselt number. On the basis of the 2-DFM, this sharp rise occurs at values of the length Reynolds number of 4,000,000, 4,500,000 and 4,500,000 in Figs. 5, 6, and 7, respectively. These values of the length Reynolds number at the start of transition are in good accord with values given in the literature for transition of a laminar boundary layer on a flat plate with zero pressure gradient.

Results for Test Combination C - Effect of Shorter Test Section

The results of using the 2-DFM for calculation of one run with test combination C are shown in Fig. 8. The value of the inlet diameter Reynolds number is 127,000, which corresponds to the highest value for test combination D shown in Fig. 7. Hence the effect of a shorter length of test section may be obtained by comparison of these two figures. Fig. 8 shows that the value of the Mach number, based on the 2-DFM, remains nearly constant. The boundary layer thickness ratio increases from 10 per cent at inlet to about 20 per cent at exit, thus duplicating the behaviour of the boundary-layer thickness ratio of

test combination D in Fig. 7. This ratio in Fig. 8 also agrees well with that computed by the Howarth formula.

The values of the local Stanton and local Nusselt numbers based on the 2-DFM, shown in Fig. 8, agree within a few per cent with the values based on the theoretical predictions for plate flow, in the region where a laminar boundary layer is present. The value of the length Reynolds number at transition from a laminar to a turbulent boundary layer lies between 3,000,000 and 4,000,000; this agrees well with the values observed for transition in the longer test section of test combination D. The main effect of the shorter length of test combination C is that the rise in values of the Stanton or Nusselt numbers during transition is never as spectacular as that found in test combination D.

PREDICTION OF DIABATIC SUPERSONIC FLOW IN A TUBE

The 2-DFM provides a means of predicting the flow behaviour for diabatic supersonic flow in the entrance region of a tube if a laminar boundary layer exists. In essence, this method of prediction was confirmed when test combination D was designed to be 50 diameters long instead of 30 for the previous test combination C. The analysis for prediction for diabatic flow is obtained by rearrangement of the basic equations given above but will be omitted here for lack of space. The analysis based on the 2-DFM is also being used to design new test sections for adiabatic and diabatic flow with an inlet Mach number near 5.

CONCLUSIONS

An approximate and simplified two-dimensional flow model for the entrance flow region of a tube yields results which are in agreement with theoretical predictions for tube flow and for plate flow with zero pressure gradient. This flow model yields a better understanding of the phenomena which occur in diabatic supersonic flow of air in a round tube than that obtained with the simple one-dimensional flow model and eliminates the inadequacies of the latter model.

The Mach numbers based on the 2-DFM are significantly larger and much more nearly constant along the test section than those based on the 1-DFM.

The thickness of the laminar boundary layer based on the 2-DFM increases slowly from tube inlet to exit but does not fill the tube cross section at exit. These values of the thickness are in good agreement with those calculated from Howarth's results.

The values of the local Stanton number based on the 2-DFM agree best with the theoretical predictions for tube flow and deviate most from those for plate flow at the lowest value of the diameter Reynolds number, namely, 33,000.

But the reverse is true at the highest value of the diameter Reynolds number, namely, 127,000 for test combination C, and 115,000 for test combination D. The agreement between measured tube values and predictions for plate flow for zero pressure gradient for these last two values of the diameter Reynolds number is within a few per cent. The values of the local Nusselt number based on the 2-DFM behave similar to the Stanton numbers in comparisons with the theoretical predictions for tube flow and plate flow but are of less value in understanding and interpreting the phenomena.

The effect of varying the length of test section from 30 to 50 diameters on the results based on the 2-DFM is negligible, except that the shorter length does not delineate transition as clearly.

The transition from a laminar to a turbulent boundary layer for diabatic supersonic flow in a tube is clearly evident in the sudden and very sharp rise of the values of the local Stanton number. The values of the length Reynolds number at the start of the transition are in excellent agreement with similar data for a laminar boundary layer on a flat plate.

ACKNOWLEDGMENTS

The assistance of W. S. Wu, P. Ashurkoff, D. M. Coleman, and Mrs. A. B. Walker in the computations is gratefully acknowledged.

This investigation is sponsored at Contract Number N5-ori-07805 by the Office of Naval Research of the United States Navy.

BIBLIOGRAPHY

1. "An Inexpensive Supersonic Wind Tunnel for Heat-Transfer Measurements Part I - Apparatus, Data, and Calculated Results for a Laminar Boundary Layer Based on a Simple One-Dimensional Flow Model," by J. Kaye, J. H. Keenan, G. A. Brown, and R. H. Shoulberg. M.I.T. Technical Report No. 6418-7, June 1, 1954.
2. "Experimental Velocity Profiles for Adiabatic Supersonic Flow of Air in a Tube," by J. Kaye, G. A. Brown, and J. J. Dieckmann, Journal of the Aeronautical Sciences, Readers' Forum, Vol. 21, 1954, pp. 203-205.
3. "Experimental Velocity Profiles for Supersonic Flow of Air in a Tube With and Without Heat Transfer," by J. Kaye, G. A. Brown, J. J. Dieckmann, and E. A. Sziklas, the Second U. S. National Congress of Applied Mechanics, University of Michigan, June, 1954.
4. "Investigation of Laminar Boundary Layer in Compressible Fluids Using the Crocco Method," by E. R. Van Driest, NACA Technical Note 2597, January, 1952.
5. "The Laminar Boundary Layer of a Steady Compressible Flow in the Entrance Region of a Tube," by T. Y. Toong, Sc.D. Thesis, M.I.T., January 1952.
6. "Concerning the Effect of Compressibility on Laminar Boundary Layers and Their Separation," by L. Howarth, Proceedings of the Royal Society of London, series A, vol. 194, 1948, pp. 16-42.
7. "Gas Tables," by J. H. Keenan and J. Kaye, John Wiley & Sons, 1948, New York, N.Y.

APPENDIX

CALCULATED RESULTS BASED ON TWO-DIMENSIONAL FLOW MODEL

This appendix contains the results based on the 2-DFM for seven runs, of which six were made with test combination D and one with C. The summary of the large amount of computation based on the 2-DFM is given in Table 1. The values given in Table 1 are based on calculations made with five or six significant figures throughout. A sample calculation is given here, in part, to indicate the method, for Run No. B-1. The original data for Run No. B-1 are taken from Table 5 of reference (1).

The values are summarized as follows:

$$G_s^* = 46.73 \text{ lbm/sec ft}^2$$

$$(Re_D^*)_s = 92,765$$

$$c_w = 0.967$$

$$w_s^* = 0.01837 \text{ lbm/sec}$$

$$w = 0.01777 \text{ lbm/sec}$$

$$A = 0.1978 \text{ in.}^2 = 0.001373 \text{ ft}^2$$

From Table 30 of the "Gas Tables" (7), corresponding to the value of (p/p_{oi}) of 0.04601 for the fourth station, it is found that

$$M_c = 2.655$$

$$T_c/T_{oi} = 0.4149$$

$$T_c = (T_c/T_{oi}) T_{oi} = 0.4149 (570.88) = 236.9^\circ \text{ F abs}$$

For the core,

$$\rho_c = \frac{p_{oi} (p/p_{oi})}{RT_c} = \frac{14.479 \times 0.04601 \times 144}{53.342 \times 236.9} = 0.007645 \text{ lbm/ft}^3$$

$$V_c = M_c \sqrt{gkRT_c} = 2.655 \sqrt{32.174 \times 1.400 \times 53.342 \times 236.9} = 2003 \text{ ft/sec}$$

$$G_c = \rho_c V_c = 0.007645 \times 2003 = 15.31 \text{ lbm/(sec ft}^2)$$

For the boundary layer,

$$V_b = V_c/2 = 1002 \text{ ft/sec}$$

From Equations (1), (2), (3), (7), (12), (13), (14), and (15)

$$A_c = \frac{q_j/2 + \sum_{n=1}^{n=j-1} q_n + w_{c_p} (T_{oi} - V_b^2/2gc_p) - (pV_{b_c_p} A)/R}{G_{c_p} (T_{oi} - V_b^2/2gc_p) - (pV_{b_c_p})/R}$$

$$q_j/2 + \sum_{n=1}^{n=j-1} q_n = q_4/2 + q_3 = 0.001482 + 0.011486 = 0.012968 \text{ Btu/sec}$$

$$\begin{aligned} (T_{oi} - V_b^2/2gc_p) &= (570.88 - 1002 \times 1002/2 \times 32.174 \times 778.3 \times 0.2399) \\ &= 487.4 \text{ } ^\circ\text{F abs} \end{aligned}$$

$$w_{c_p} (T_{oi} - V_b^2/2gc_p) = 0.01777 \times 0.2399 \times 487.4 = 2.077 \text{ Btu/sec}$$

$$G_{c_p} (T_{oi} - V_b^2/2gc_p) = 15.31 \times 0.2399 \times 487.4 = 1791 \text{ Btu/sec ft}^2$$

$$pV_{b_c_p}/R = 14.479 \times 0.04601 \times 144 \times 1002 \times 0.2399/53.342 = 435.1 \text{ Btu/sec ft}^2$$

$$pV_{b_c_p} A/R = 435.1 \times 0.001373 = 0.5976 \text{ Btu/sec}$$

$$A_c = \frac{0.012968 + 2.077 - 0.5976}{1791 - 435.1} = 0.001101 \text{ ft}^2$$

$$A_b = A - A_c = 0.001373 - 0.001101 = 0.000272 \text{ ft}^2$$

$$w_c = G_c A_c = 15.31 \times 0.001101 = 0.01687 \text{ lbm/sec}$$

$$w_b = w - w_c = 0.01777 - 0.01687 = 0.000902 \text{ lbm/sec}$$

$$\delta/a = 1 - (1/a) \sqrt{A_c/\pi} = 1 - (24/0.5018) \sqrt{0.001101/\pi} = 0.1045$$

From Equations (3) and (7)

$$\begin{aligned} T_b &= pV_b A_b / R w_b = 14.579 \times 0.04601 \times 144 \times 1002 \times 0.000272 / 53.342 \times 0.000902 \\ &= 547.3 \text{ } ^\circ\text{F abs} \end{aligned}$$

$$\rho_b = \frac{p_{oi} (p/p_{oi})}{RT_b} = \frac{14.579 \times 0.4601 \times 144}{53.342 \times 547.3} = 0.003308 \text{ lbm/ft}^3$$

$$G_b = \rho_b V_b = 0.003308 \times 1002 = 3.314 \text{ lbm/sec ft}^2$$

The remaining quantities are calculated as follows:

$$T_{aw} = (T_{aw}/T_{oi}) T_{oi} = 0.940 \times 570.88 = 536.6 \text{ } ^\circ\text{F abs}$$

$$h = q/A' (t_w - t_{aw}) = 974.7/(672.2 - 536.6) = 7.19 \text{ Btu/hr } ^\circ\text{F ft}^2$$

$$hD/\lambda_c = 7.19 \times 0.5018/12 \times 3600 \times 0.1942 \times 10^{-5} = 43.0$$

$$hL/\lambda_c = (hD/\lambda_c) (L/D) = 43.0 \times 4.484 = 193$$

$$h/c_p G_c = 7.19/0.2399 \times 15.31 \times 3600 = 5.44 \times 10^{-4}$$

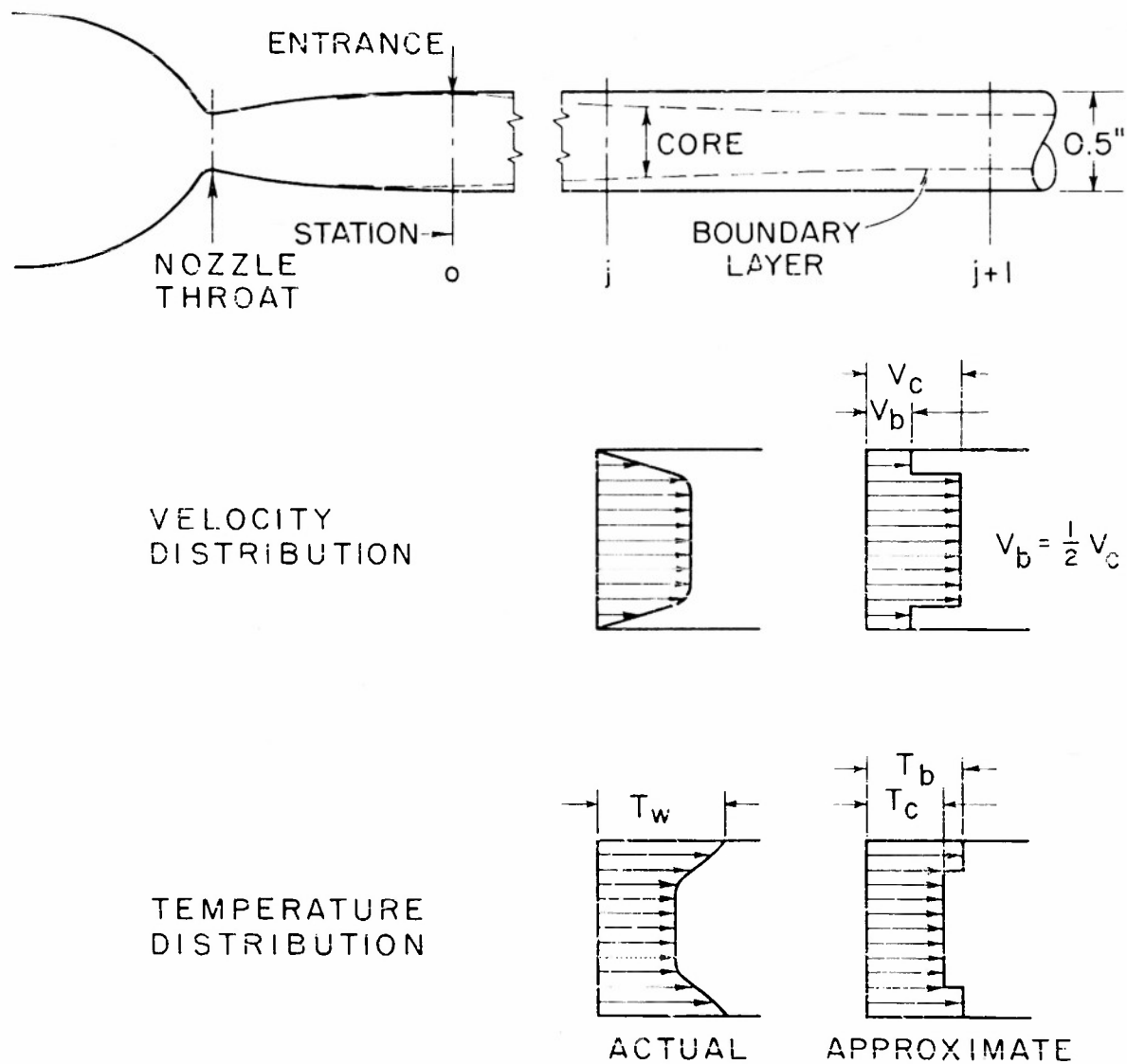
$$\text{Re}_{Lc} = \frac{G_c D (L/D)}{\mu_c} = \frac{15.31 \times 0.5018 \times 4.484}{12 \times 0.620 \times 10^{-5}} = 463,000$$

The calculations are continued by repeating the above steps for Station numbers 5, 6, 7, etc.

TABLE 1

CALCULATIONS BASED ON TWO-DIMENSIONAL FLOW MODE

Station		P ₀₁		T ₀₁		T ₀₂		T ₀₃		T ₀₄		T ₀₅		T ₀₆		T ₀₇		T ₀₈		T ₀₉		T ₁₀		T ₁₁		T ₁₂		T ₁₃		T ₁₄		T ₁₅		T ₁₆		T ₁₇		T ₁₈		T ₁₉		T ₂₀		T ₂₁		T ₂₂		T ₂₃		T ₂₄		T ₂₅		T ₂₆		T ₂₇		T ₂₈		T ₂₉		T ₃₀		T ₃₁		T ₃₂		T ₃₃		T ₃₄		T ₃₅		T ₃₆		T ₃₇		T ₃₈		T ₃₉		T ₄₀		T ₄₁		T ₄₂		T ₄₃		T ₄₄		T ₄₅		T ₄₆		T ₄₇		T ₄₈		T ₄₉		T ₅₀		T ₅₁		T ₅₂		T ₅₃		T ₅₄		T ₅₅		T ₅₆		T ₅₇		T ₅₈		T ₅₉		T ₆₀		T ₆₁		T ₆₂		T ₆₃		T ₆₄		T ₆₅		T ₆₆		T ₆₇		T ₆₈		T ₆₉		T ₇₀		T ₇₁		T ₇₂		T ₇₃		T ₇₄		T ₇₅		T ₇₆		T ₇₇		T ₇₈		T ₇₉		T ₈₀		T ₈₁		T ₈₂		T ₈₃		T ₈₄		T ₈₅		T ₈₆		T ₈₇		T ₈₈		T ₈₉		T ₉₀		T ₉₁		T ₉₂		T ₉₃		T ₉₄		T ₉₅		T ₉₆		T ₉₇		T ₉₈		T ₉₉		T ₁₀₀		T ₁₀₁		T ₁₀₂		T ₁₀₃		T ₁₀₄		T ₁₀₅		T ₁₀₆		T ₁₀₇		T ₁₀₈		T ₁₀₉		T ₁₁₀		T ₁₁₁		T ₁₁₂		T ₁₁₃		T ₁₁₄		T ₁₁₅		T ₁₁₆		T ₁₁₇		T ₁₁₈		T ₁₁₉		T ₁₂₀		T ₁₂₁		T ₁₂₂		T ₁₂₃		T ₁₂₄		T ₁₂₅		T ₁₂₆		T ₁₂₇		T ₁₂₈	
---------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	-----------------	--	------------------	--	------------------	--	------------------	--	------------------	--	------------------	--	------------------	--	------------------	--	------------------	--	------------------	--	------------------	--	------------------	--	------------------	--	------------------	--	------------------	--	------------------	--	------------------	--	------------------	--	------------------	--	------------------	--	------------------	--	------------------	--	------------------	--	------------------	--	------------------	--	------------------	--	------------------	--	------------------	--	------------------	--	------------------	--



SIMPLIFIED TWO-DIMENSIONAL FLOW MODEL
FIG. 1

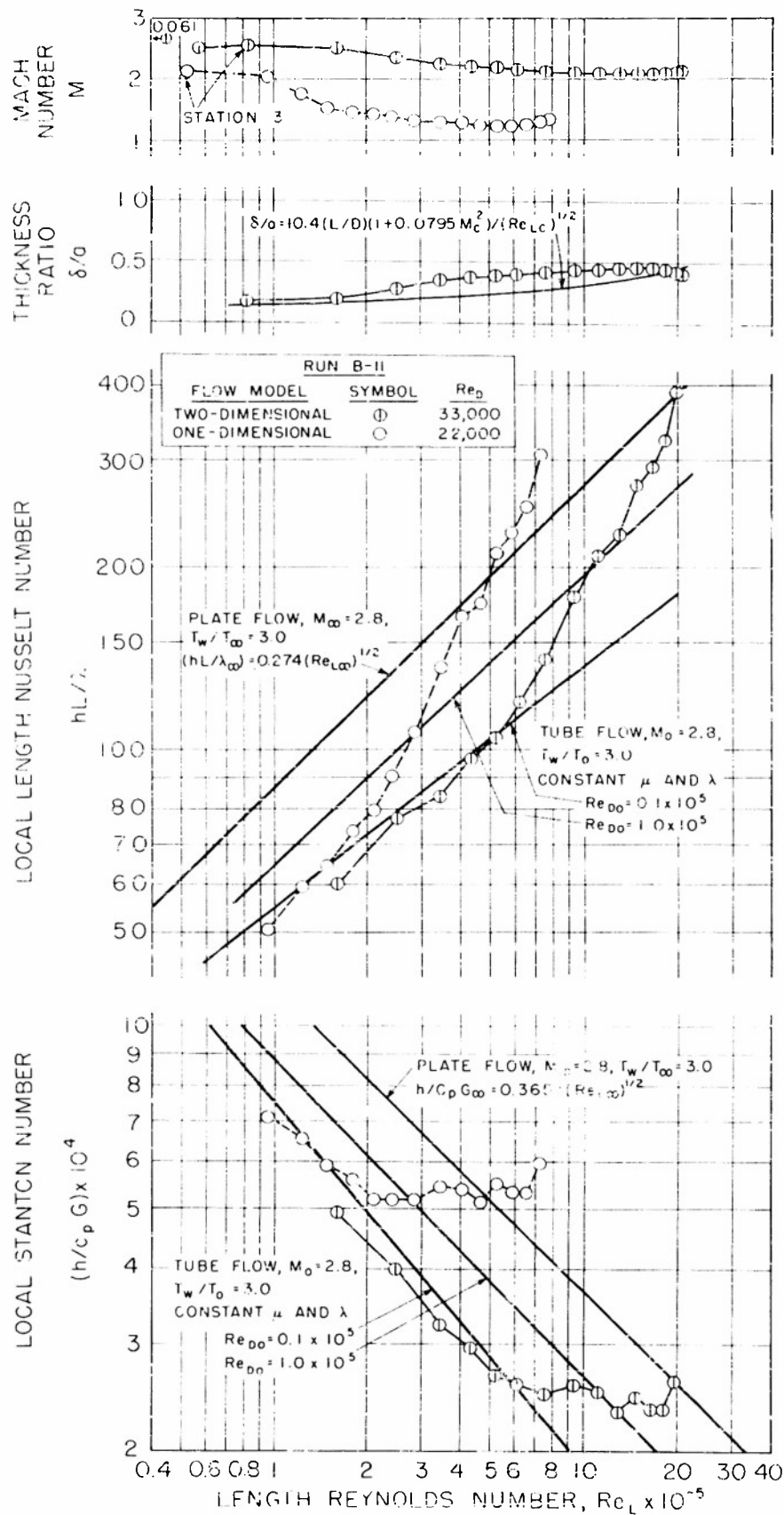


FIGURE 2

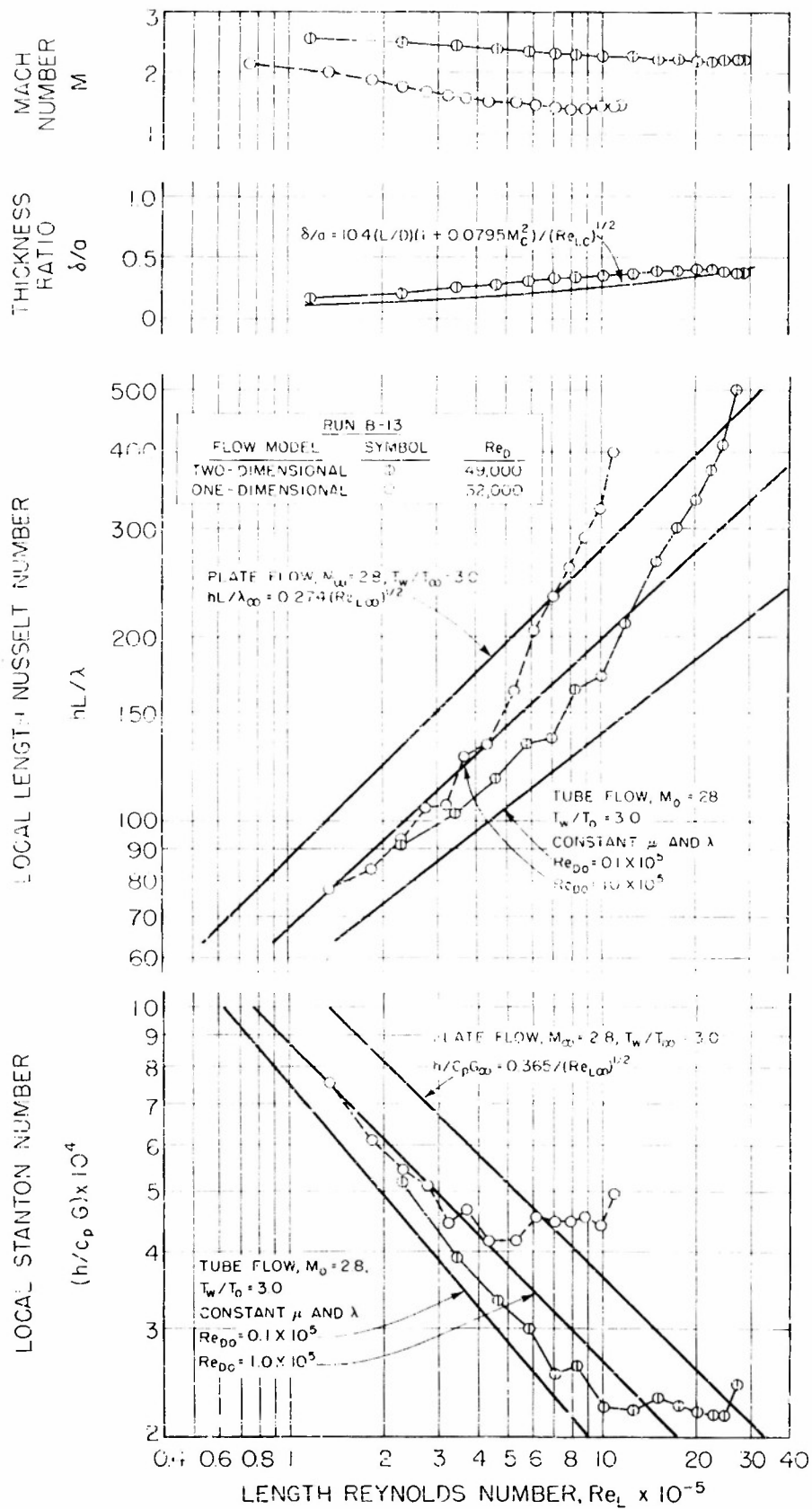


FIGURE 3

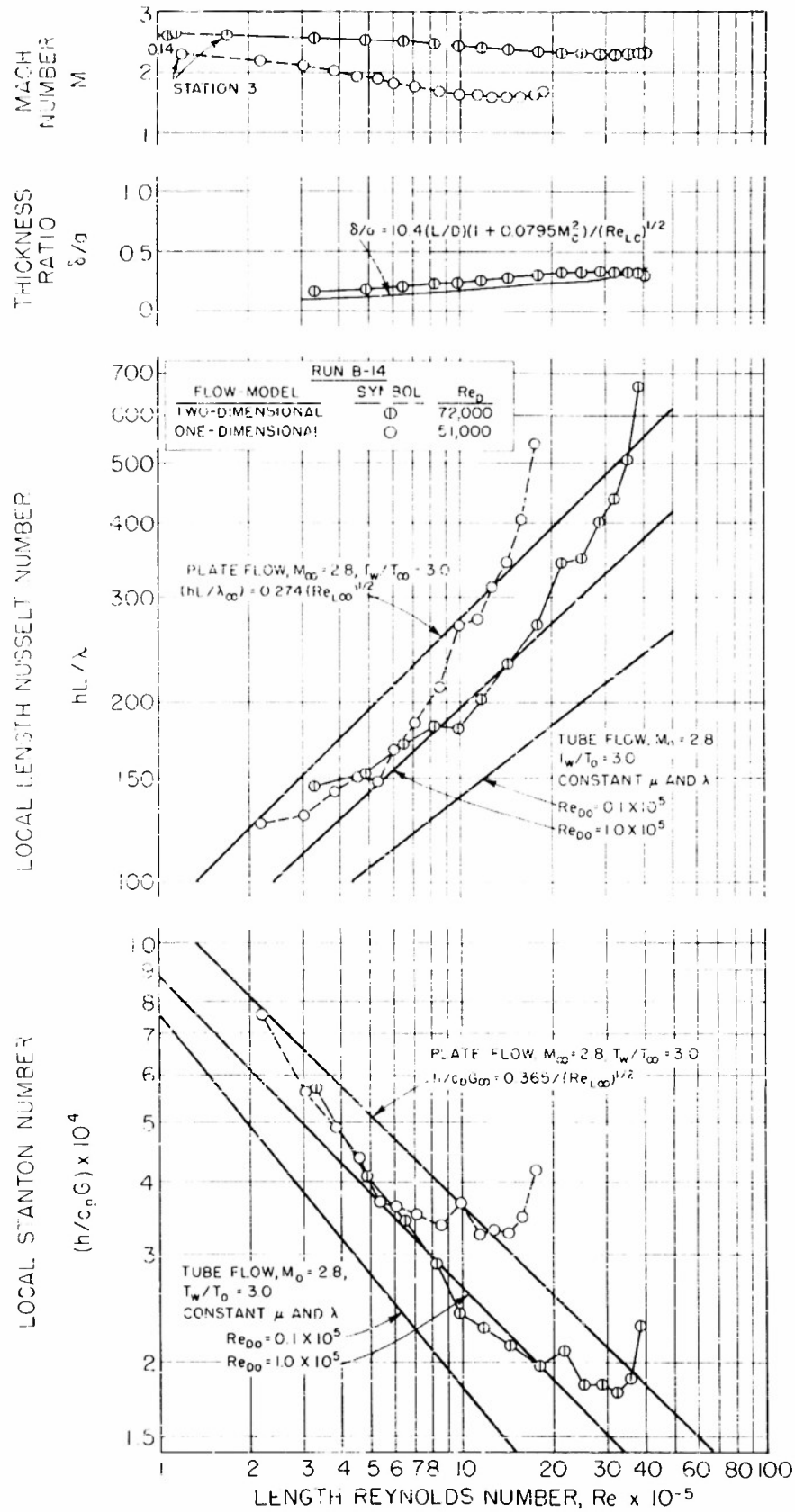


FIGURE 4

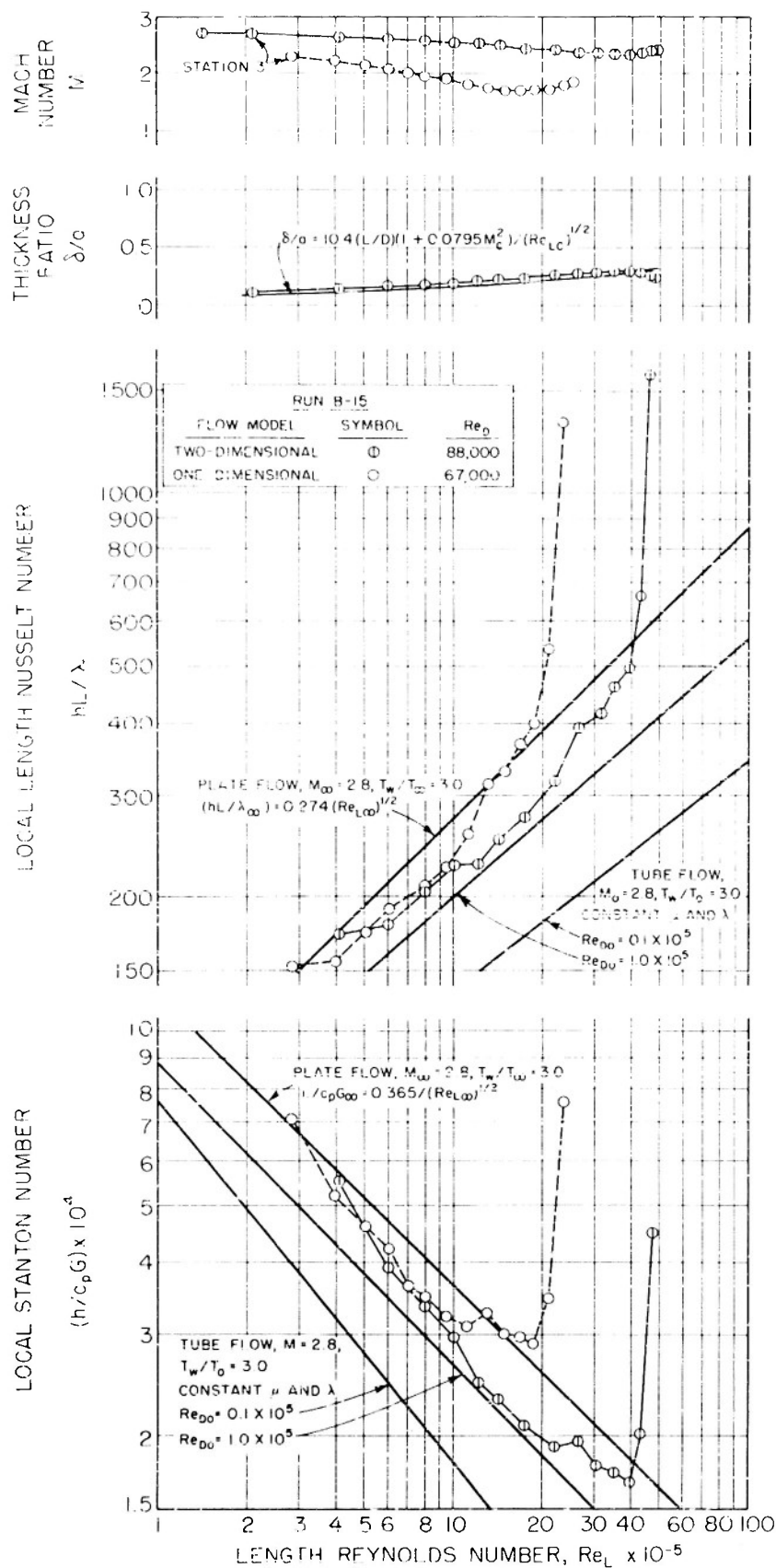


FIGURE 5

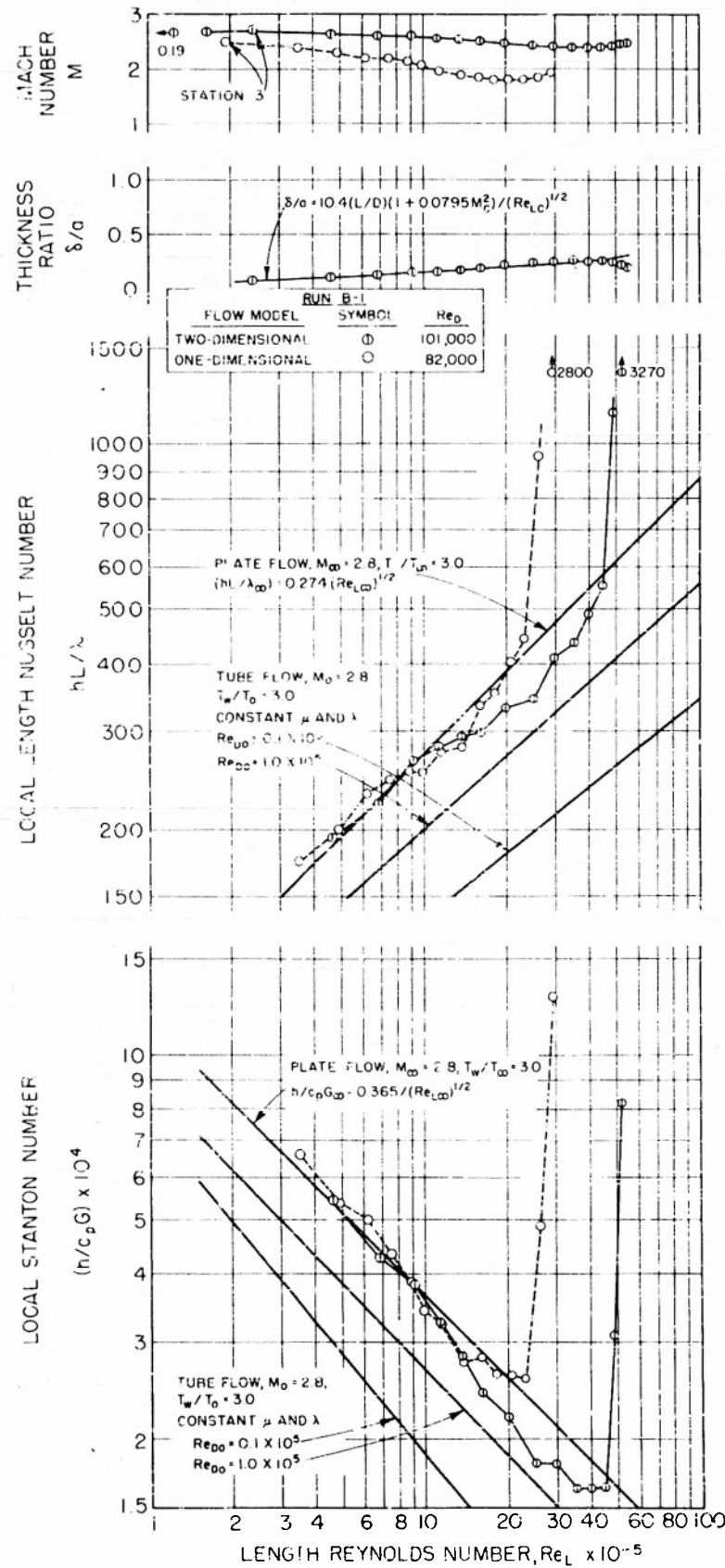


FIGURE 6

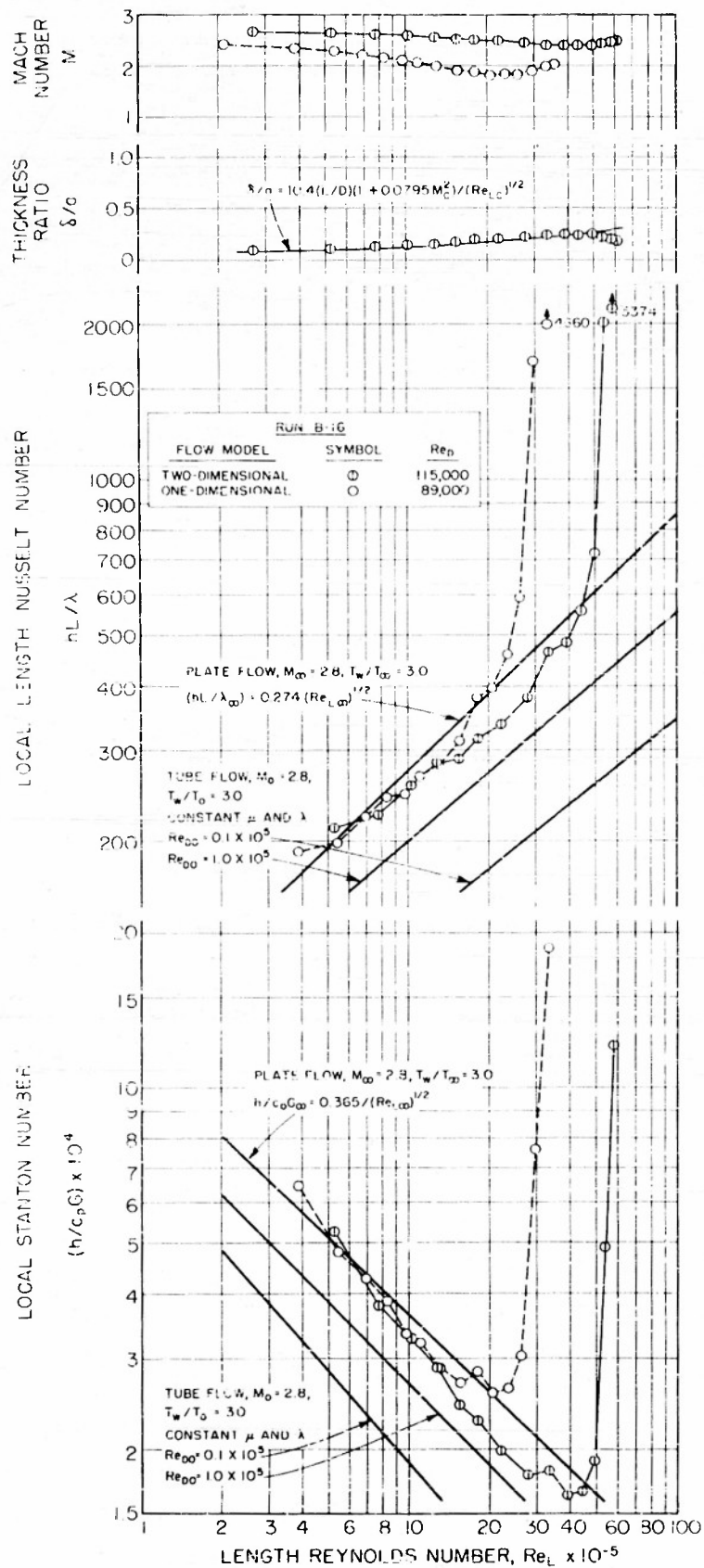


FIGURE 7

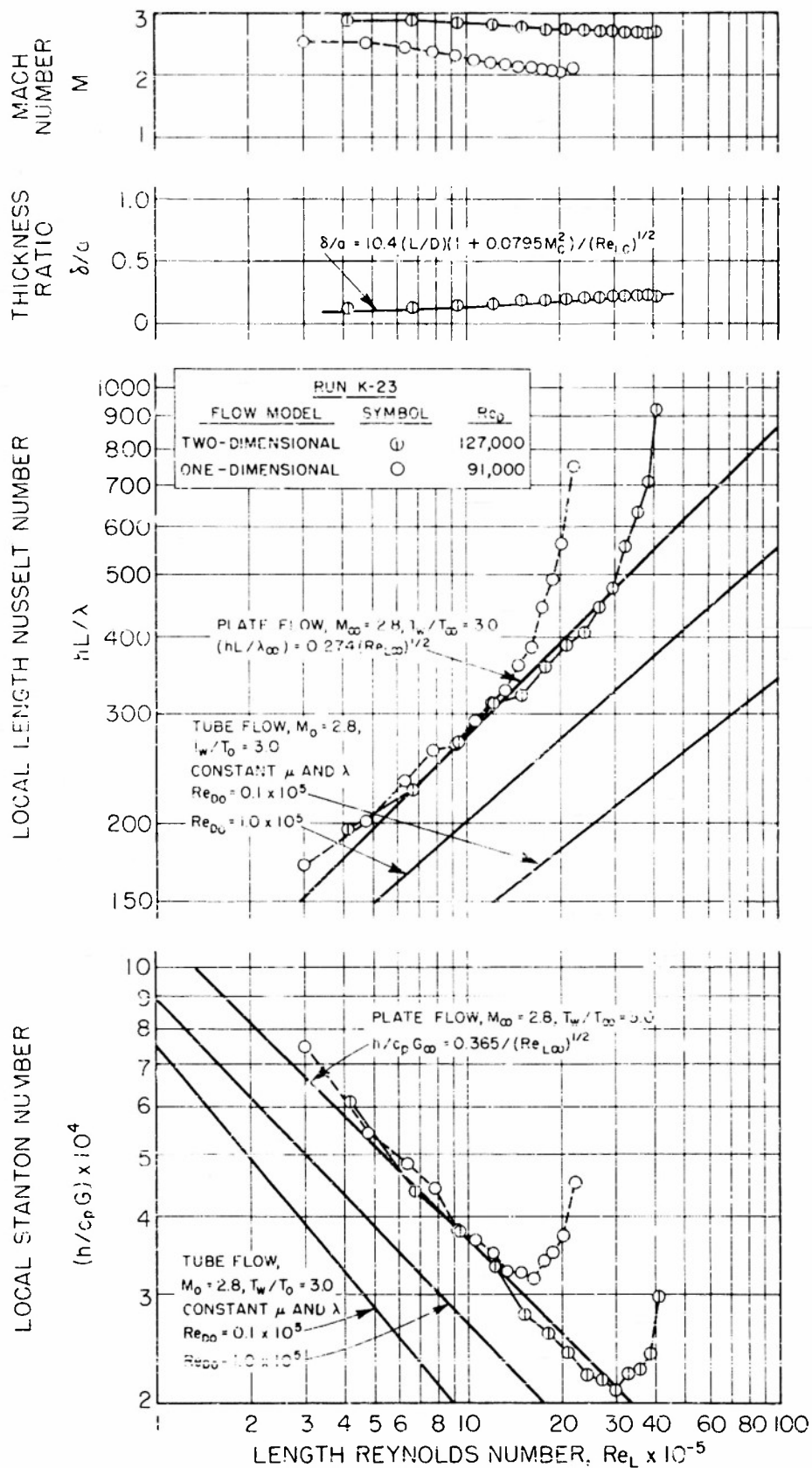


FIGURE 6

REVISED DISTRIBUTION LIST FOR UNCLASSIFIED
TECHNICAL REPORTS ISSUED UNDER
CONTRACT N5ori-07805 PROJECT NR 061-028

Chief of Naval Research Department of the Navy Washington 25, D. C. Attn: Code 438 (2)	Commanding Officer Naval Ordnance Laboratory White Oak, Silver Spring, Md. Attn: Dr. R. K. Lobb (1)
Commanding Officer Office of Naval Research Branch Office 150 Causeway Street Boston 10, Massachusetts (2)	Chief, Bureau of Ships Department of the Navy Washington 25, D. C. Attn: Research Division (1)
Commanding Officer Office of Naval Research Branch Office 346 Broadway New York 13, New York (1)	Commanding Officer and Director David Taylor Model Basin Washington 7, D. C. Attn: Aeromechanics Division (1) Hydrodynamics Division (1)
Commanding Officer Office of Naval Research Branch Office Tenth Floor The John Crerar Library Bldg. 86 East Randolph Street Chicago 1, Illinois (1)	Directorate of Intelligence Headquarters, U.S. Air Force National Defense Building Washington 25, D. C. Attn: Doc. and Dissem. Br.(AFOIR-DD) (1)
Commanding Officer Office of Naval Research Branch Office 1000 Geary Street San Francisco 1, California (1)	Armed Services Technical Information Agency Document Service Center Knott Building Dayton 2, Ohio (5)
Commanding Officer Office of Naval Research Branch Office 1030 E. Green Street Pasadena 1, California (1)	Commanding General, Hdqs. Air Research and Development Command Office of Scientific Research P. O. Box 1395 Baltimore, 18, Maryland Attn: Fluid Mechanics Div. (1)
Commanding Officer Office of Naval Research Navy #100, FPO New York, New York (2)	Director, Research and Development Department of the Army National Defense Building Washington 25, D. C. (1)
Chief, Bureau of Aeronautics Department of the Navy Washington 25, D. C. Attn: Research Division (1)	Commanding General Office of Ordnance Research Department of the Army Washington 25, D. C. (1)
Chief, Bureau of Ordnance Department of the Navy Washington 25, D. C. Attn: Code Re9a (1)	Ballistic Research Laboratories Department of the Army Aberdeen Proving Ground, Maryland Attn: Mr. J. Sternberg (1)

Director of Research National Advisory Committee for Aeronautics 1724 F Street, Northwest Washington 25, D. C. (1)	University of Michigan Department of Aeronautical Engineering East Engineering Building Ann Arbor, Michigan Attn: Dr. Arnold Kuethe (1) Prof. E. W. Conlon (1)
Director National Advisory Committee for Aeronautics Langley Aeronautical Laboratory Langley Field, Virginia (1)	California Institute of Technology Guggenheim Aeronautical Laboratory Pasadena 4, California Attn: Dr. Hans W. Liepmann (1) Prof. Lester Lees (1) Dr. P. A. Lagerstrom (1)
Director National Advisory Committee for Aeronautics Langley Aeronautical Laboratory Langley Field, Virginia Attn: Dr. A. Busemann (1)	Director National Bureau of Standards Department of Commerce Washington 25, D. C. Attn: Dr. G. B. Schubauer (1)
Director National Advisory Committee for Aeronautics Ames Aeronautical Laboratory Moffett Field, California (1)	Polytechnic Institute of Brooklyn Department of Aeronautical Engineering and Applied Mechanics 99 Livingston Street Brooklyn 2, New York Attn: Dr. A. Ferri (1)
Director National Advisory Committee for Aeronautics Ames Aeronautical Laboratory Moffett Field, California Attn: Dr. D. R. Chapman (1)	Brown University Division of Engineering Providence 12, Rhode Island Attn: Dr. P. F. Maeder (1)
Director National Advisory Committee for Aeronautics Lewis Flight Propulsion Laboratory 21000 Brookpark Road Cleveland 11, Ohio (1)	California Institute of Technology Jet Propulsion Laboratory Pasadena 4, California Attn: Mr. F. E. Goddard (1)
Director National Advisory Committee for Aeronautics Lewis Flight Propulsion Laboratory 21000 Brookpark Road Cleveland 11, Ohio Attn: Dr. J. C. Evvard (1)	University of California Engineering Research Projects Berkeley 4, California Attn: Dr. S. A. Schaaf (1)
The Johns Hopkins University Department of Aeronautical Engineering Baltimore 18, Maryland Attn: Dr. F. H. Clauser (1)	University of California Department of Engineering Los Angeles 24, California Attn: Dean L.M.K. Boelter (1)
The Johns Hopkins University Applied Physics Laboratory 8621 Georgia Avenue Silver Spring, Maryland Attn: Dr. F. N. Frankiel (1)	Cornell Aeronautical Laboratory 1455 Genessee Street Buffalo 21, New York Attn: Mr. A. H. Flax (1)
	Cornell University Graduate School of Aeronautical Engineering Ithaca, New York Attn: Dr. W. R. Sears (1)

University of Delaware
Department of Chemical Engineering
Newark, Delaware
Attn: Dr. Kurt Wohl (1)

Harvard University
Department of Applied Physics
and Engineering Science
Cambridge 38, Massachusetts
Attn: Prof. H. W. Emmons (1)

Massachusetts Institute of Technology
Guided Missiles Committee
Cambridge 39, Massachusetts
Attn: Supervisor, Tech. Reports (1)

Massachusetts Institute of Technology
Department of Aeronautical Engineering
Cambridge 39, Massachusetts
Attn: Dr. M. Finston (1)

Massachusetts Institute of Technology
Department of Mechanical Engineering
Cambridge 39, Massachusetts
Attn: Prof. J. H. Keenan (1)
Prof. A. H. Shapiro (1)

Princeton University
Department of Aeronautical Engineering
Princeton, New Jersey
Attn: Prof. S. M. Bogdonoff (1)

Purdue University
School of Mechanical Engineering
Lafayette, Indiana
Attn: Dr. N. J. Zucrow (1)

University of Texas
Defense Research Laboratory
500 East 24th Street
Austin, Texas
Attn: Dr. M. J. Thompson (1)

Rensselaer Polytechnic Institute
Department of Aeronautical Engineering
Troy, New York
Attn: Dr. R. P. Harrington (1)

University of Maryland
Institute for Fluid Dynamics
and Applied Mathematics
College Park, Maryland
Attn: Dr. S. I. Pai (1)

Case Institute of Technology
Department of Mechanical Engineering
Cleveland, Ohio
Attn: Prof. G. Kuerti (1)

Stanford University
Department of Mechanical Engineering
Stanford, California
Attn: Prof. A. L. London (1)

Armed Services Technical Information Agency

Because of our limited supply, you are requested to return this copy WHEN IT HAS SERVED YOUR PURPOSE so that it may be made available to other requesters. Your cooperation will be appreciated.

AD 45690

NOTICE: WHEN GOVERNMENT OR OTHER DRAWINGS, SPECIFICATIONS OR OTHER DATA ARE USED FOR ANY PURPOSE OTHER THAN IN CONNECTION WITH A DEFINITELY RELATED GOVERNMENT PROCUREMENT OPERATION, THE U. S. GOVERNMENT THEREBY INCURS NO RESPONSIBILITY, NOR ANY OBLIGATION WHATSOEVER; AND THE FACT THAT THE GOVERNMENT MAY HAVE FORMULATED, FURNISHED, OR IN ANY WAY SUPPLIED THE SAID DRAWINGS, SPECIFICATIONS, OR OTHER DATA IS NOT TO BE REGARDED BY IMPLICATION OR OTHERWISE AS IN ANY MANNER LICENSING THE HOLDER OR ANY OTHER PERSON OR CORPORATION, OR CONVEYING ANY RIGHTS OR PERMISSION TO MANUFACTURE, USE OR SELL ANY PATENTED INVENTION THAT MAY IN ANY WAY BE RELATED THERETO.

Reproduced by
DOCUMENT SERVICE CENTER
KNOTT BUILDING, DAYTON, 2, OHIO

UNCLASSIFIED